Theory of Computation: Assignment 1 Solutions

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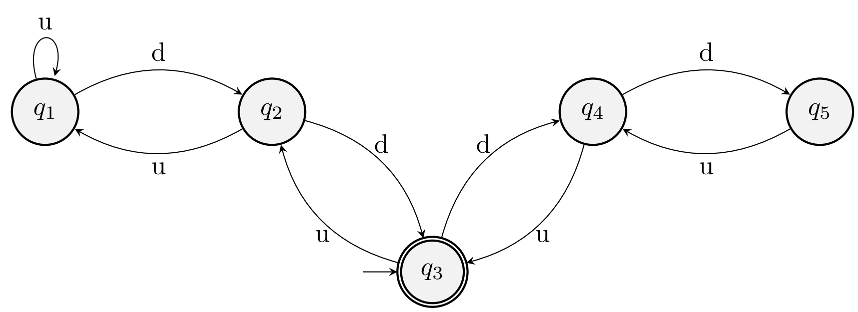
1. (a) For the function problem, we take two numbers (a, b) as input, and we output the greatest common divisor of a and b.

   (b) For the decision problem, we take three numbers (a, b, k) as input. We output ACCEPT if $\gcd(a, b) = k$ and REJECT otherwise.

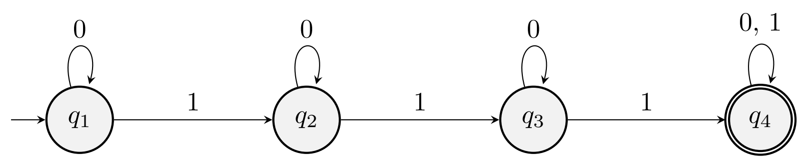
   (c) Suppose we are given (a, b, k) as input. We use the function problem to compute $d = \gcd(a, b)$. We output ACCEPT if $d = k$ and REJECT otherwise

   (d) Suppose we are given (a, b) as input. Assume without loss of generality that $a \leq b$. For $k = a, a - 1, \ldots, 1$, we use the decision problem to check if $\gcd(a, b) = k$. The first time we receive ACCEPT, we stop and output k.

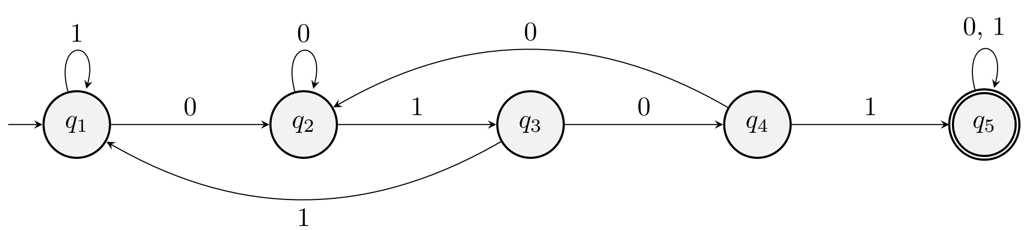
2. The state diagram for M is as follows:



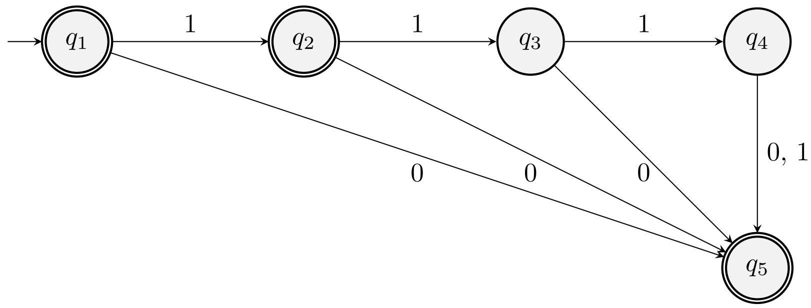
3. (a) $L = \{w | w \text{ contains at least three 1s}\}$



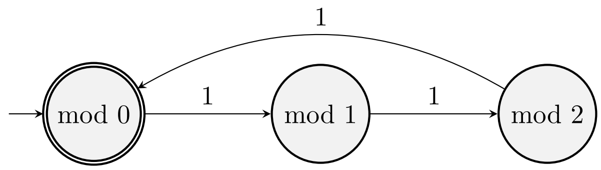
   (b) $L = \{w | w \text{ contains a the substring 0101}\}$



   (c) $L = \{w | w \text{ is any string except 11 and 111}\}$



4. (a) The state diagram for L_3 is as follows:



(b) For a given value of n, we can design a DFA $M_n = (Q_n, \Sigma, \delta_n, q_{s_n}, F_n)$ to recognize L_n. The formal definition of M_n is as follows:

- $Q_n = \{0, 1, \dots, n-1\}$ i.e. all possible remainders modulo n. The state keeps track of what our current remainder is.
- $\Sigma = \{1\}$
- If $k < n-1$, then $\delta(k, 1) = (k+1)$. Otherwise, $\delta(n-1, 1) = 0$. Basically, every time we read a new digit, the remainder gets incremented by 1. However, if we are at remainder $n-1$ and we read another digit, we reset the counter back to 0.
- $q_s = 0$ - initially we start our counter at a remainder of 0
- $F = \{0\}$ - to accept, we need to be at remainder 0