Theory of Computation: Assignment 1

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Due Thursday, 01/27/2022 at 11:59 pm (50 points)

1. Given two integers a and b, the **greatest common divisor (gcd)** of a and b is the largest number that is a factor of both a and b. For example:

   - $\gcd(5, 15) = 5$
   - $\gcd(16, 20) = 4$
   - $\gcd(9, 16) = 1$
   - $\gcd(100, 100) = 100$

   The problem of calculating the gcd of two numbers important in number theory and cryptography.

   (a) (5 points) Express the gcd problem as a function problem and a decision problem

   (b) (10 points) Show that if we had a crystal ball to solve the function problem, we could design an algorithm to solve the decision problem (and vice-versa)

2. (5 points) This problem is taken from Sipser exercise 1.3. Let M be a DFA whose formal description is $(\{q_1, q_2, q_3, q_4, q_5\}, \{u, d\}, \delta, q_3, \{q_3\})$. The transition function δ is described by the following table:

	u	d
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_5

   Draw the state diagram for M.

3. The following problems are taken from Siper exercises 1.6b, c and h. For each language below, give a state diagram for a DFA that recognizes that language. In each case the alphabet is $\Sigma = \{0, 1\}$.

   (a) (5 points) $L = \{w | w \text{ contains at least three 1s}\}$

   (b) (5 points) $L = \{w | w \text{ contains the substring 0101}\}$

   (c) (5 points) $L = \{w | w \text{ is any string except 11 and 111}\}$

4. Consider the alphabet $\Sigma = \{1\}$. Strings from this alphabet include $1, 111, 11111111111$, etc. This is called a unary alphabet, and languages on this alphabet are called unary numbers.

   For any positive integer $n > 0$, define the language $L_n \subseteq \Sigma^*$ to be the set of strings whose length is divisible by n. For example, $L_3 = \{\epsilon, 111, 111111, 111111111, \dots\}$.

   (a) (5 points) Draw the state diagram for L_3

   (b) (10 points) Prove that for all n, L_n is a regular language. For full credit, give the formal definition that describes how you would construct the correct DFA for a given value of n.