Theory of Computation: Assignment 5

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Due Thursday, 02/24/2022 at 11:59 pm

1. (5 points) This problem is taken from exercise 1.30 in Sipser. Describe the error in the following "proof" that 0^*1^* is not a regular language.

   > AFSOC 0^*1^* is regular. Let p be the pumping length for 0^*1^* given by the pumping lemma. Choose s to be the string 0^p1^p. You know that s is a member of 0^*1^*, and it can be split into xyz such that $|xy| \leq p$ and $|y| > 0$. Since $|xy| \leq p$, the xy part contains only 0s. Because $|y| > 0$, when we pump up we will increase the number of 0s, and then the 0s and 1s won't be equal, meaning the new string will not be in the language. Thus you have a contradiction. So 0^*1^* is not regular.

2. The following problems are taken from exercise 1.29ab and problem 1.46a in Sipser. Prove that the following languages are not regular

   (a) (5 points) $L = \{0^n1^n2^n | n \geq 0\}$

   (b) (5 points) $L = \{www | w \in \{a, b\}^*\}$. In this language, each string contains three consecutive copies of the same (smaller) string.

   (c) (5 points) $L = \{0^n1^m0^n | m, n \geq 0\}$

3. This problem is based on problem 1.46b in Sipser. A palindrome is a string that reads the same forwards and backwards, such as *001100*, *ABBA*, *racecar*, and *lonely tylenol*.

   (a) (5 points) Prove that the language $L = \{w | w \in \{0, 1\}^*, w \text{ is a palindrome}\}$ is not regular.

   (b) (10 points) Prove that the language $L = \{w | w \in \{0, 1\}^*, w \text{ is } \underline{\text{not}} \text{ a palindrome}\}$ is not regular. For this problem, you *must* use the pumping lemma to get credit. (**Hint:** try using the string $0^p10^{p!+p}$, where $p! = 1 \times 2 \times \cdots \times (p-1) \times p$.)

   (c) (5 points) Now give an alternate proof that $L = \{w | w \in \{0, 1\}^*, w \text{ is } \underline{\text{not}} \text{ a palindrome}\}$ is not regular. Your proof should *not* use the pumping lemma; instead, it should use the result from part (a), as well as the fact that regular languages are closed under complement.

4. (10 points) Recall that a unary language is a language on an alphabet with just one symbol. Let $\Sigma = \{1\}$. Prove that the following language is not regular:

   $$\text{PRIMES} = \{w | w \in \Sigma^*, \text{ the length of w is a prime number}\}$$

   Examples of strings in PRIMES include $11, 111, 1111111, 1^{53}$, etc.

   (**Hint:** You can use pretty much any string $s = 1^q$ where q is prime and bigger than the pumping length p. If y contains i 1s, then x and z contain $q - i$ 1s. Figure out how many times to pump y so that the total number of 1s is no longer prime.)