Theory of Computation: Assignment 6

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Due Thursday, 03/17/2022 at 11:59 pm (50 points)

1. (5 points) Recall that in class we proved that L is Turing-recognizable if and only if there is an enumerator that enumerates it. In the backwards direction, we gave an algorithm to enumerate all the strings if we know the language is recognizable. Explain why the following (simpler) algorithm would not have worked

   1. Go through every possible string $s_1, s_2, \ldots$ and carry out the first step of M's computation on each s_i

   2. Carry out the second step of M's computation on each s_i

   3. Repeat the process of going through each string s_i and carrying out the "next" step of computation. Whenever any one string gets accepted, print it out.

2. (5 points) This problem is taken from problem 3.7 in Sipser. Explain why the following is not a legitimate description for a Turing machine:

   1. M takes a polynomial equation $\langle p \rangle$ over variables $x_1, x_2, \ldots x_n$ as input

   2. Try all possible settings of $x_1, x_2, \ldots x_n$ to integer values

   3. Evaluate p on all of these settings

   4. If any settings evaluate to 0, accept $\langle p \rangle$. Otherwise, reject $\langle p \rangle$.

3. (10 points) Prove if a language L is regular, then it is Turing-recognizable. For full credit on this problem, you must use the formal definition of a TM as a 7-tuple in your proof. (**Hint:** Show how to convert the DFA for L into a TM. Try to simulate the DFA reads characters one at a time and updates its states.)

4. This problem is taken from exercise 3.8 in Sipser. Give tape-level descriptions for Turing machines to recognize the following languages on the alphabet $\Sigma = \{0, 1\}$

   (a) (5 points) $L = \{w | w$ contains an equal number of 0s and 1s$\}$

   (b) (5 points) $L = \{w | w$ contains twice as many 0s as 1s$\}$

   (c) (5 points) $L = \{w | w$ does <u>not</u> contain an equal number of 0s and 1s$\}$

5. (5 points) Briefly explain why the previous two problems prove that Turing machines are strictly more powerful than DFAs.

6. (10 points) This problem is taken from problem 3.11 in Sipser. A **doubly infinite Turing machine** is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is the same, except that the head never encounters an "end" to the tape as it moves leftward. Prove that a language is Turing-recognizable if and only if it is recognized by a doubly infinite TM. For full credit, make sure your proof includes <u>both</u> directions.