Theory of Computation: Assignment 7

Arjun Chandrasekhar

Due Thursday, 03/24/2021 at 11:59 pm (50 points)

1. (a) (5 points) This problem is taken from problem 3.15b in Sipser. Prove that Turing-decidable languages are closed under concatenation.
   (b) (5 points) This problem is taken from problem 3.16b in Sipser. Prove that Turing-recognizable languages are closed under concatenation.
2. (10 points) This problem is based on exercise 4.2 in Sipser. Consider the problem of determining whether a DFA and a regex are equivalent to each other (i.e. recognize the same language). Express this as a formal language, and then prove that it is decidable.
3. (10 points) This problem is based on exercise 4.3 in Sipser. Prove that the following language is decidable

   $$\mathrm{ALL}_{\mathrm{DFA}} = \{\langle D\rangle | D \text{ is a DFA, } L(D) = \Sigma^*\}$$

   We are given a DFA description as input, and we want to check if the DFA accepts *every* string.
4. (10 points) Show that the following language is decidable

   $$L = \{\langle D\rangle | D \text{ is a DFA that does not accept any string containing more than four 1's}\}$$

   Hint: the set of strings that contain more than four 1's is a regular language.
5. (10 points) This problem is based on problem 4.20 in Sipser. Let A and B be disjoint languages - that is, $A \cap B = \emptyset$. We say that a language C **separates** A and B if $A \subseteq C$ and $B \subseteq \overline{C}$. We say that A and B are **recursively separable** if there is a decidable language C that separates A and B.

   Suppose that $\overline{A}$ and $\overline{B}$ are Turing-recognizable. Prove that A and B are recursively separable. (**Hint:** convince yourself that every string must be in either $\overline{A}$ or $\overline{B}$. Use this to design a machine M that accepts strings in A and rejects strings in B.)