Theory of Computation: Assignment 8

Arjun Chandrasekhar

Due 03/31/2022 at 11:59 pm (30 points)

1. For these two problems, we will formally define a **homework assignment** to be a string the English alphabet (including spaces, punctuation, etc.) just think of all of the characters in the assignment writeup as comprising one big long string A, which we call the assignment.

   (a) (10 points) Prove that the set of all *finite-length* homework assignments is countable. (**Hint:** consider all assignments with 0 characters; then 1 character, 2 characters, and so on.)

   (b) (10 points) Prove that the set of all *infinite-length* homework assignments is uncountable. (**Hint:** create a new homework assignment that disagrees with all the other assignments in at least one position.)

2. (10 points) Consider the following language language

$$\overline{\text{HALT}} = \{\langle M, w\rangle | M \text{ loops on } w\}$$

   This is the complement of HALT, usually called/pronounced "co-HALT". Prove that $\overline{\text{HALT}}$ is undecidable. Your proof mimic the diagonalization proof that we used in class for HALT.