Theory of Computation: Assignment 9

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Due 04/07/2022 at 11:59 pm (50 points)

1. (10 points) Consider the following language

$$\text{INF}_{\text{TM}} = \{\langle M \rangle | L(M) \text{ is finite}\}$$

We are given a TM description as input, and we want to determine if the machine can accept infinitely many strings. Prove that INF_{TM} is undecidable. (**Hint:** reduce from A_{TM})

2. (10 points) Consider the following language.

$$\text{DIS}_{\text{TM}} = \{\langle M_1, M_2 \rangle | L(M_1) \cap L(M_2) = \emptyset\}\}$$

We are given two machines, and we want to check if the machines are disjoint - that is, they don't recognize any of the same strings.

Prove that DIS_{TM} is undecidable. (**Hint:** reduce from E_{TM}. Use the fact that $\emptyset$ is the only language that is disjoint from Σ^*.)

3. (10 points) Consider the following language

$$L = \{\langle M, D \rangle | M \text{ is a TM, } D \text{ is a DFA, } L(M) = L(D)\}$$

We are given a TM description and a DFA description, and we want to determine if the two machines are equivalent. Note that this is different from EQ_{TM} because one of the input machines is a DFA.

Prove that L is undecidable. (**Hint:** reduce from ALL_{TM}).

4. (10 points) This problem is taken from problem 5.22 in Sipser. Prove that a language L is Turing-recognizable if and only if $L \leq_m \text{A}_{\text{TM}}$. For full credit make sure your proof includes *both* directions.

5. (a) (5 points) Prove that if $L \leq_m \overline{L}$ then $\overline{L} \leq_m L$

   (b) (5 points) Prove that L is Turing-recognizable and $L \leq_m \overline{L}$ then L is decidable