Deterministic Finite Automata

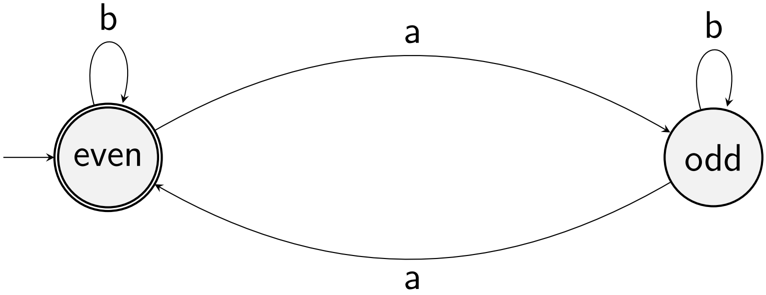
Arjun Chandrasekhar

Deterministic Finite Automata

A model of computation that uses a constant amount of memory to recognize a language.

- Several states
  - One *start state*
  - Some states are labelled as *accept states*
- Transitions (arrows) between states
  - Each transition is labelled by a character from the alphabet

Deterministic Finite Automata



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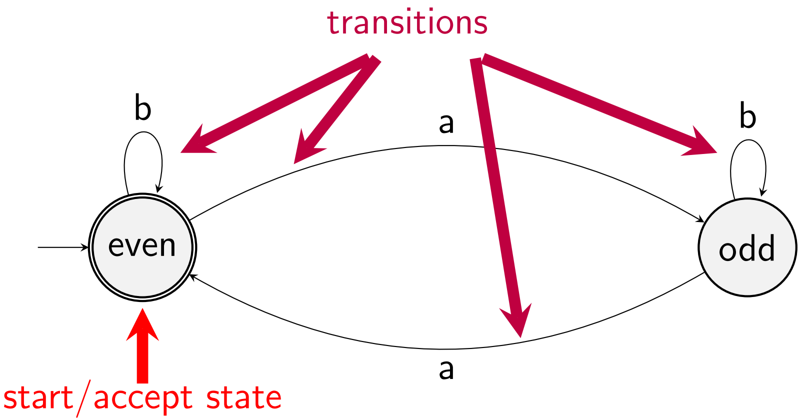

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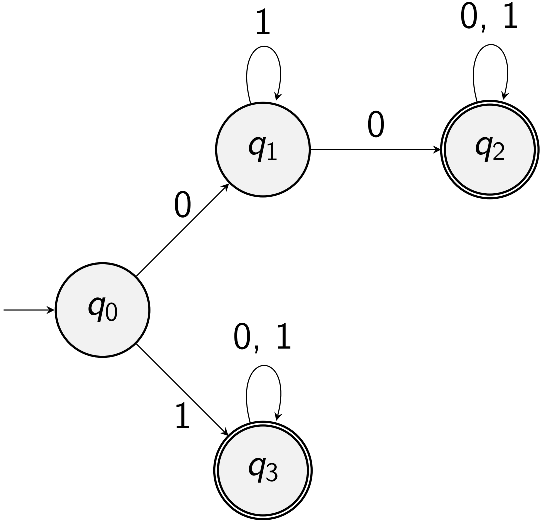

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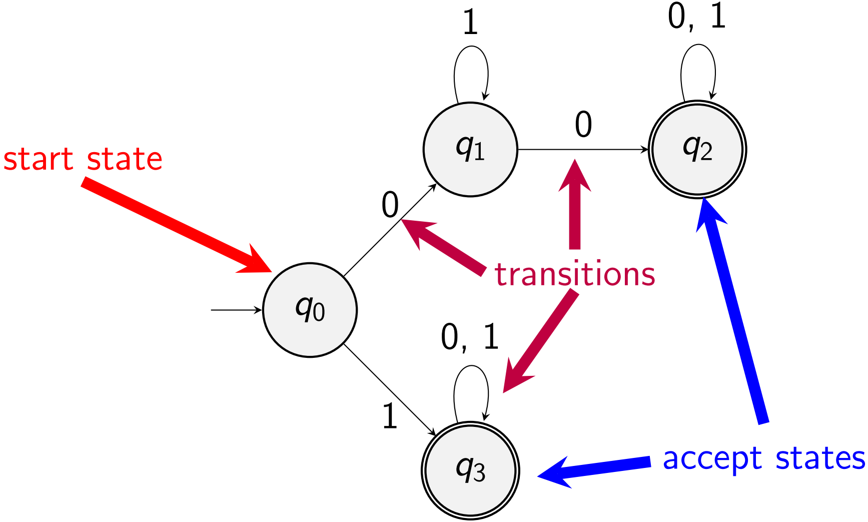

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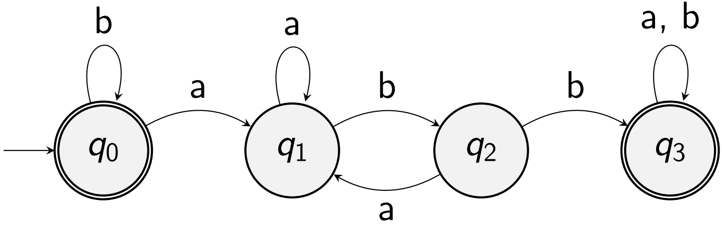


What is the start state of this DFA?

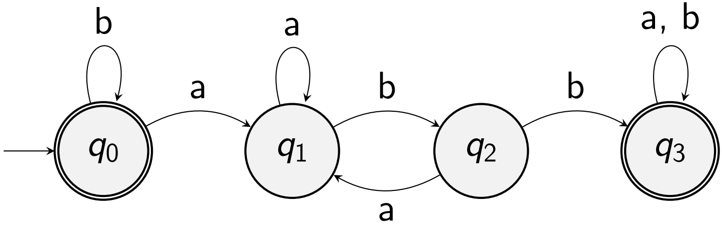
A. q_0

B. q_1

C. q_2

D. q_3

Deterministic finite automata



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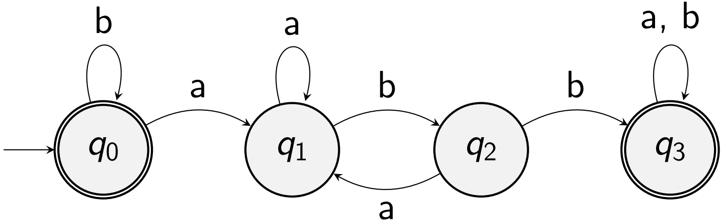


What are the accept states of this DFA?

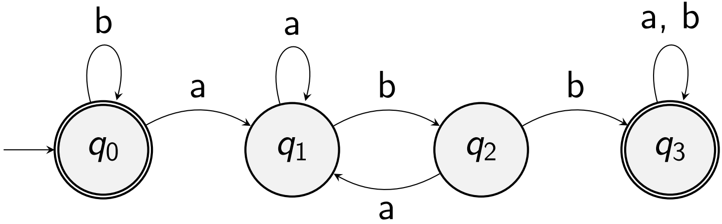
A. q_0

B. q_1

C. q_2

D. q_3

Deterministic finite automata



What are the accept states of this DFA?

A. q_0 ✓

B. q_1

C. q_2

D. q_3 ✓

Computation on a DFA

- Start in the start state
- Scan the symbols in the input one by one
- For each symbol σ scaned:
  - Go to the next state by following the arrow with the label σ
- After scanning all of the input, if the DFA is in an accept state, the input is accepted. Otherwise the input is rejected

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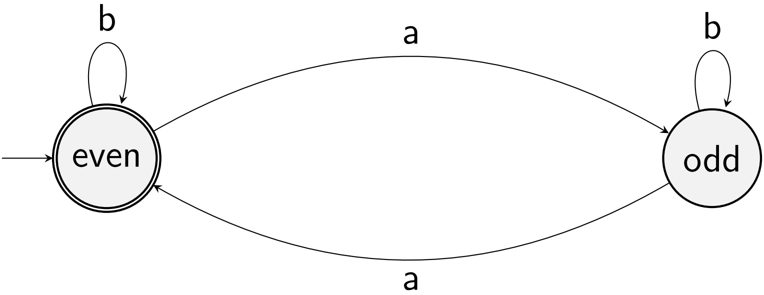
What does this DFA do on input aaaa?



Deterministic finite automata

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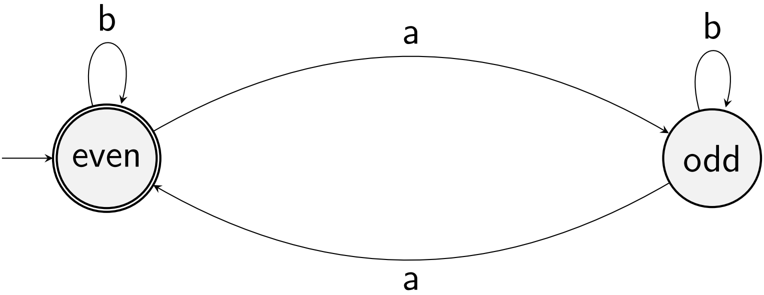
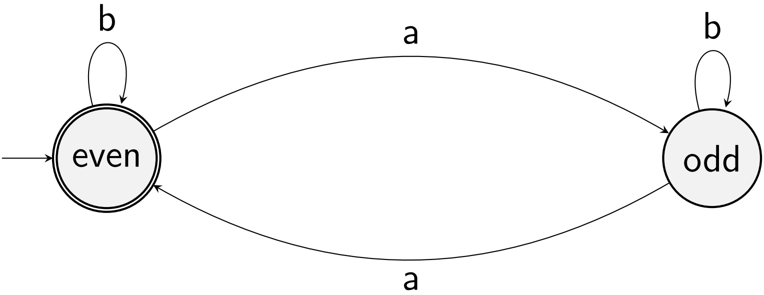


$$\text{even} \xrightarrow{a} \text{odd} \xrightarrow{a} \text{even} \xrightarrow{a} \text{odd} \xrightarrow{a} \text{even} \rightarrow \mathsf{ACCEPT}$$

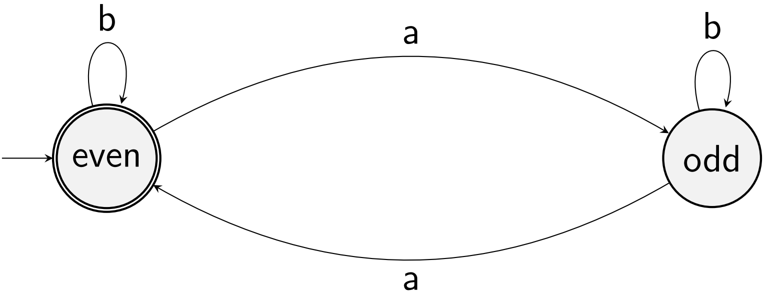
Deterministic finite automata

What does this DFA do on input b?



Deterministic finite automata

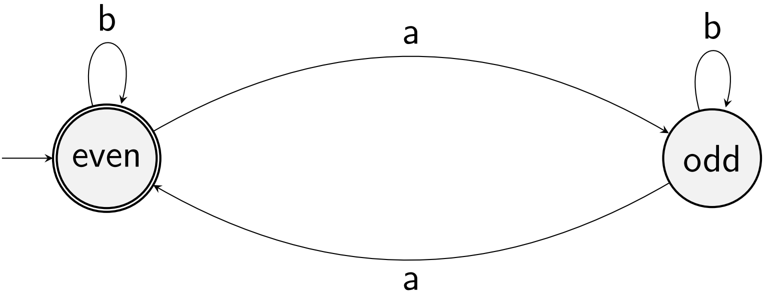
What does this DFA do on input b?



$$\text{even} \xrightarrow{b} \text{even} \rightarrow \text{ACCEPT}$$

Deterministic finite automata

What does this DFA do on input abb?



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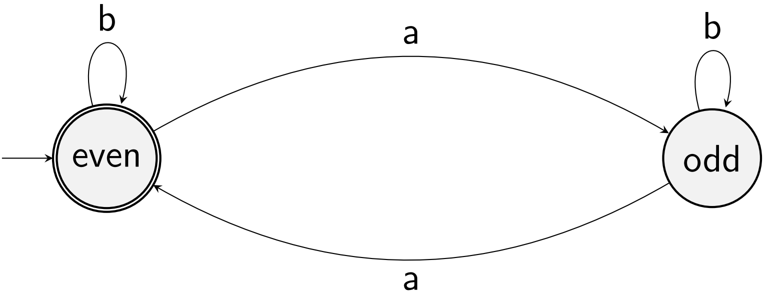
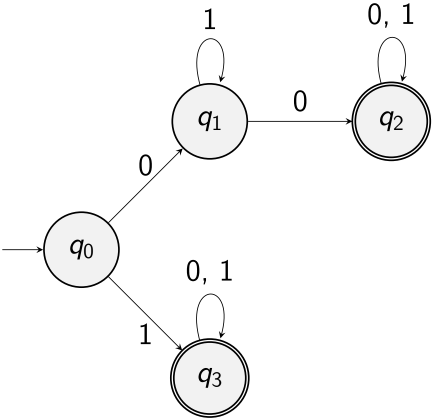
What does this DFA do on input abb?



$$\text{even} \xrightarrow{a} \text{odd} \xrightarrow{b} \text{odd} \xrightarrow{b} \text{odd} \rightarrow \text{REJECT}$$

Deterministic Finite Automata

What does this DFA do on input 0110?



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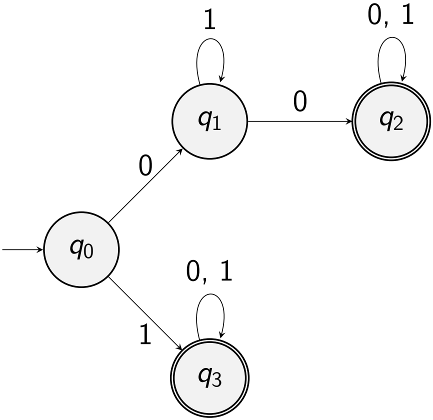
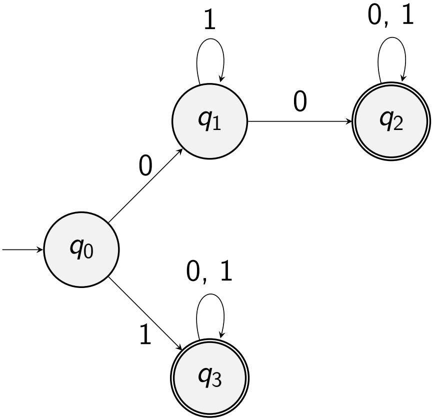
What does this DFA do on input 0110?



$$q_0 \xrightarrow{0} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \rightarrow \text{ACCEPT}$$

Deterministic Finite Automata

What does this DFA do on input 1011?



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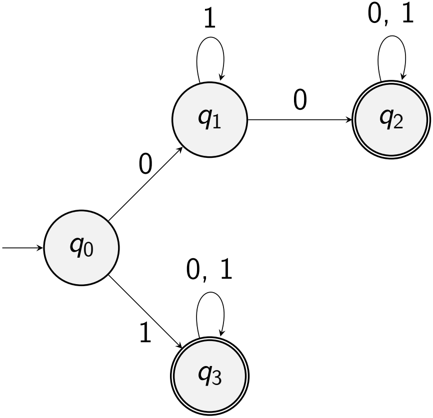
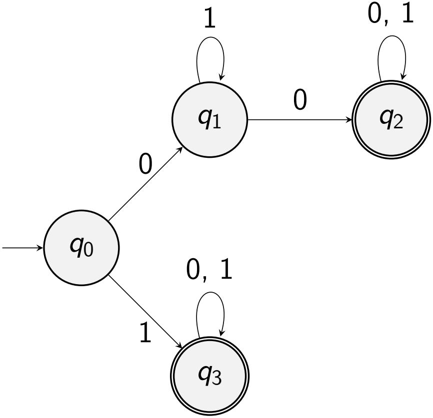
What does this DFA do on input 1011?



$$q_0 \xrightarrow{1} q_3 \xrightarrow{0} q_3 \xrightarrow{1} q_3 \xrightarrow{1} q_3 \rightarrow \text{ACCEPT}$$

Deterministic Finite Automata

What does this DFA do on input 0111?



Deterministic Finite Automata

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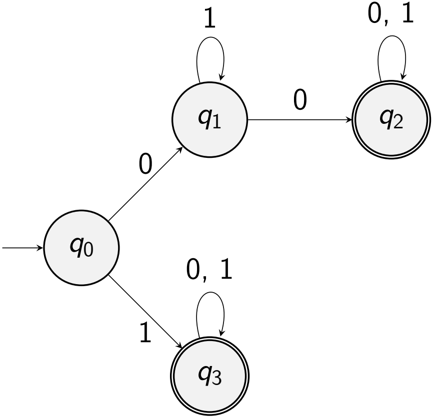
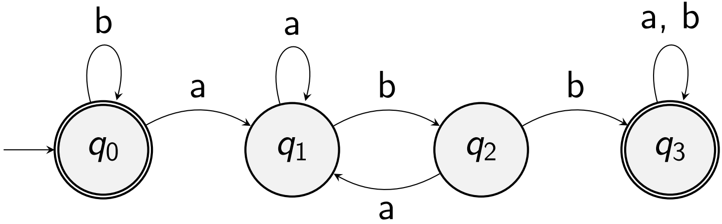


$$q_0 \xrightarrow{0} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \rightarrow \text{REJECT}$$

Deterministic Finite Automata



What strings are accepted by this DFA?

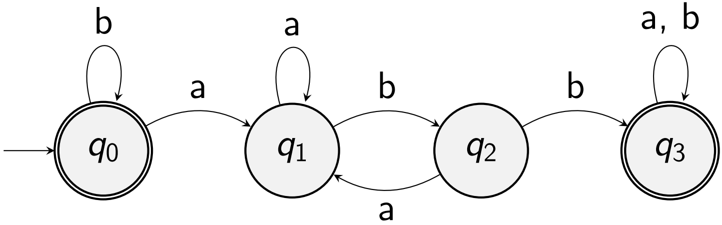
A. ϵ

B. *abbbb*

C. *bbbba*

D. $b^{100} = \underbrace{b \ldots b}_{100}$

Deterministic Finite Automata



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Formal definition of a DFA

Def: A **Deterministic Finite Automata (DFA)** is a 5-tuple $(Q, \Sigma, \delta, q_s, F)$

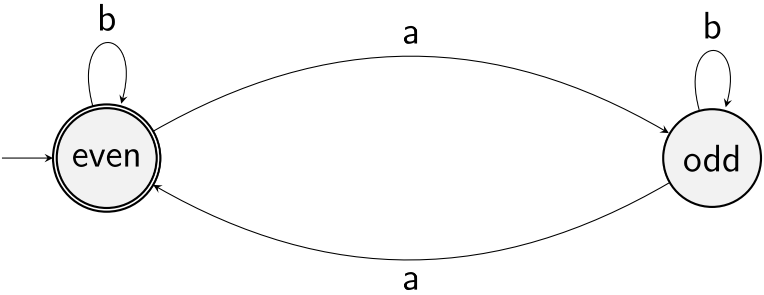
- Q - The set of states in the DFA
- Σ - The alphabet of characters that the DFA can read
- $\delta : Q \times \Sigma \to Q$ - the transition function
  - The rules for how to move between states
  - **Input:** Current state & next symbol
  - **Output:** Next state
- q_s - the starting state
- F - the set of accepting states

Formal defintion DFA computation

We say a DFA $D = (Q, \Sigma, q_s, \delta, F)$ accepts a string $w = \sigma_1\sigma_2 \ldots \sigma_n$ if there exists a sequence of states $q_0, q_1, \ldots, q_n$ such that:

- $q_0 = q_s$ (start in the start state)
- $q_i = \delta(q_{i-1}, \sigma_i)$ for all i (all transitions are valid)
- $q_n \in F$ (finish in an accept state)

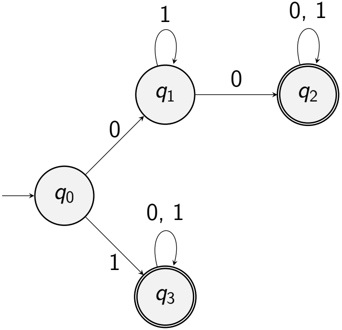
Formal definition example



- $Q = \{\text{even}, \text{odd}\}$
- $\Sigma = \{a, b\}$
- $q_s = \text{even}$
- $F = \{\text{even}\}$
- $\delta =$

	a	b
even	odd	even
odd	even	odd

Write the formal definition



- $Q = \{q_0, q_1, q_2, q_3\}$
- $\Sigma = \{0, 1\}$
- $q_s = q_0$
- $F = \{q_2, q_3\}$
- $\delta =$

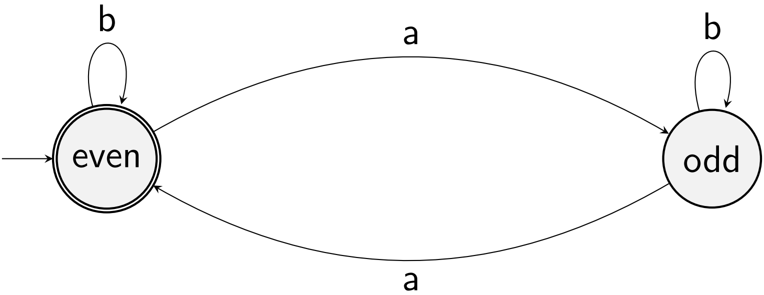
	0	1
q_0	q_1	q_3
q_1	q_2	q_1
q_2	q_2	q_2
q_3	q_3	q_3

The language of a DFA

- Let D be a DFA
- We say a DFA D **recognizes** a language L if for all $w \in \Sigma^*$:
  - If $w \in L$, then D accepts w
  - If $w \notin L$, then D rejects w
  - Do we ever have to worry about D looping?
- The **language of D**, denoted $L(D)$ is the (unique) language that D recognizes - that is, the set of all strings that D accepts

The language of a DFA

What the language of this DFA?



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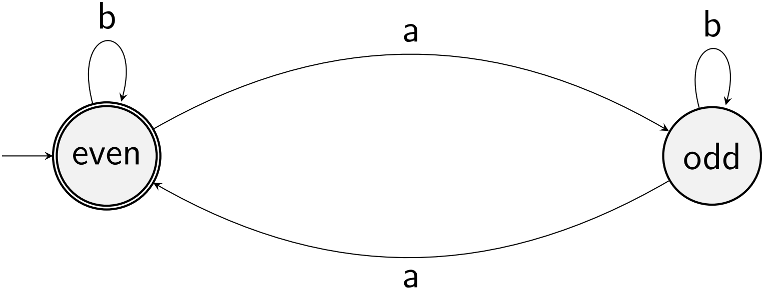

$L(D) = \{\text{w} \mid \text{w has an even number of a's}\}$

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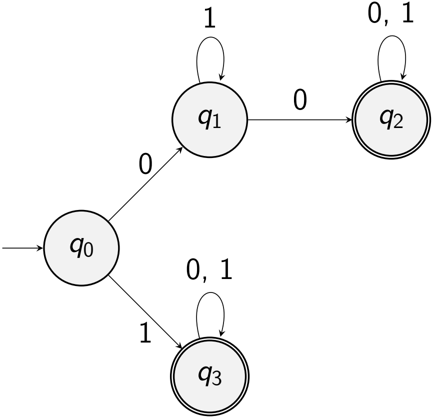

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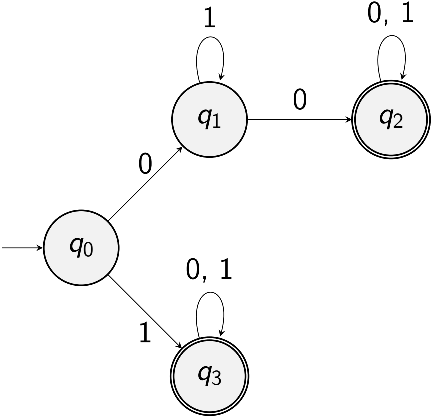

$L(D) = \{\text{w} \mid \text{w starts with 1 or has at least two 0's}\}$

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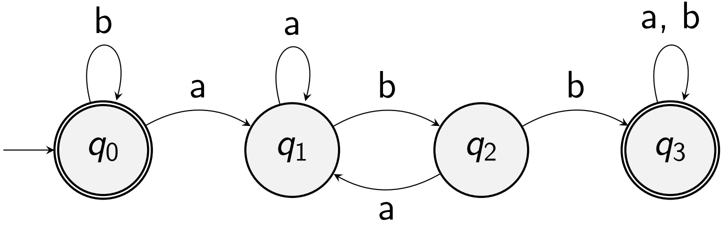

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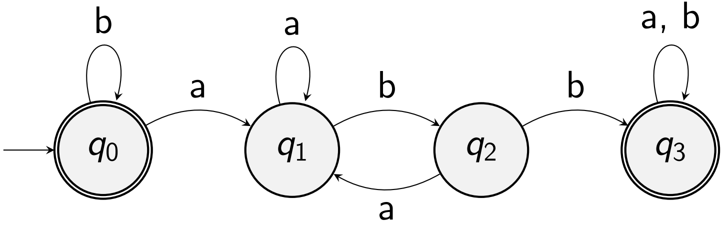

$L(Q) = \{\text{w} \mid \text{w contains only b's or contains abb}\}$

Deterministic Finite Automata

Some comments:

- A transition must be defined for every symbol in the alphabet, for every state in the DFA.
- There can be more than one accepting state.
- The set of languages that can be recognized by some DFA is called the **regular languages**.
  - A language L is regular if and only if some DFA D recognizes L

Design a DFA

Let $\Sigma = \{a, b\}$. Let's show that the following language is regular:

$$\mathrm{L} = \{\mathrm{w} \mid \mathrm{w} \text{ ends with b}\}$$

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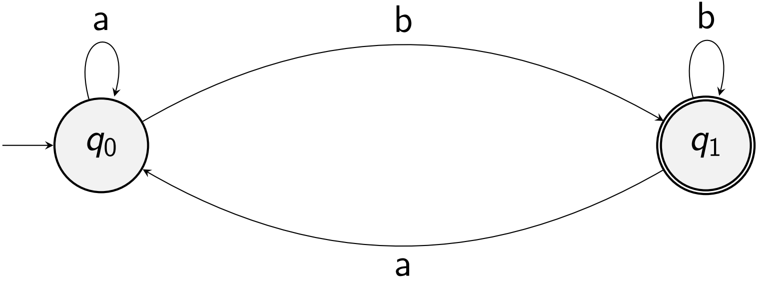

Design a DFA

Let $\Sigma = \{0, 1\}$. Let's show that the following language is regular:

$$\mathsf{L} = \{\mathsf{w} \mid \mathsf{w}\ \text{starts and ends with the same symbol}\}$$

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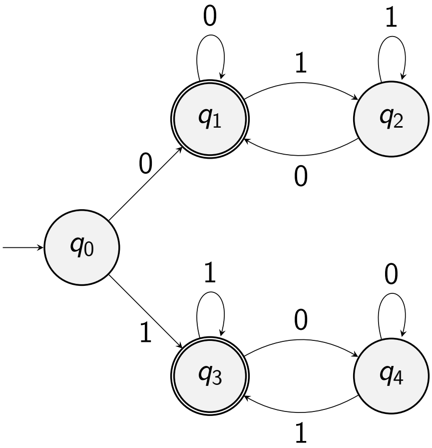

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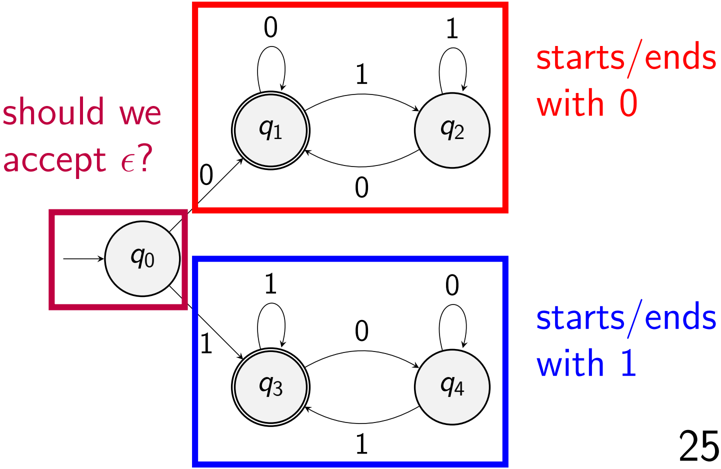

Design a DFA

Let $\Sigma = \{+, -, 0, 1, \ldots, 9\}$. Let's show that the following language is regular:

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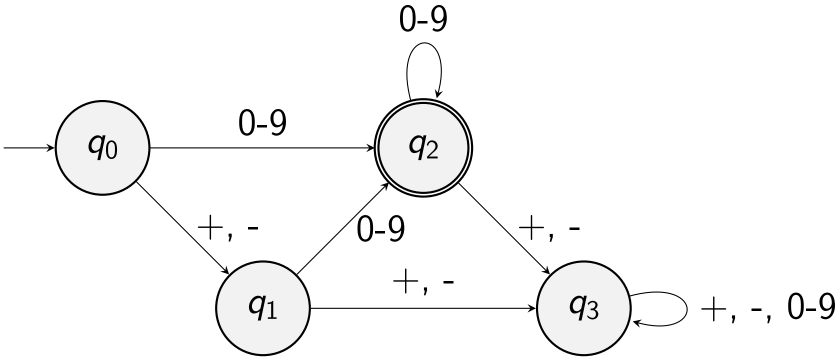

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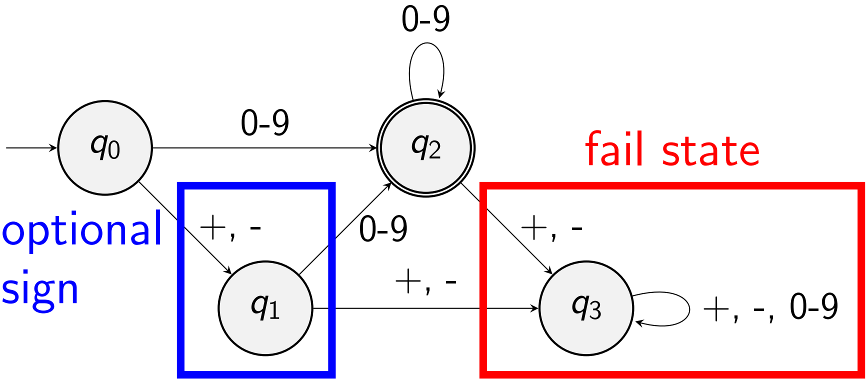

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