Theory of Computation
Decidable Languages

Arjun Chandrasekhar

State Machines as Input and Output

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- A Turing Machine can take a DFA as input

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  - This is what you did on assignment 1!

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  - Think about how the java compiler takes your source code as it its input

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  - This is what you did on assignment 2!
- A Turing machine can take a TM as input
  - Think about how the java compiler takes your source code as it its input
- A Turing machine can create other TMs
  - Think about how the compiler uses your source code to create executable files

Decidable DFA Properties

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Given an arbitrary DFA D, can we write a program to determine certain properties about it?

- Does D accept a particular string w?
- Does D accept *anything*? Does it reject everything?
- Does D accept every string that contains 111?

The language A_{DFA}

Consider the following language

$A_{DFA} = \{\langle D, w\rangle | D \text{ is a DFA description}, w \in L(D)\}$

What is the input to the decision problem associated with this language?

A) A description of a DFA

B) A regular language

C) A DFA description, and an input string

D) A DFA that is designed to recognize the string w

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C) Design a TM that designs a DFA D that accepts w

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  5. Otherwise, M rejects $\langle D, w\rangle$

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This is an example of "reducing" or "converting" one problem (A_{NFA}) to another one (A_{DFA}) that we already know how to solve

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Recap

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- We can write a program to test if an arbitrary DFA accepts an arbitrary string

Decidable DFA Properties

Recap

- We can write a program to test if an arbitrary DFA accepts an arbitrary string
- We can do the same with NFAs, regexes, and anything equivalent

The language $\mathrm{E}_{\mathrm{DFA}}$

Consider the following language

$$\mathrm{E}_{\mathrm{DFA}} = \{\langle D \rangle | D \text{ is a DFA}, L(D) = \emptyset\}$$

What should we do to show that L is decidable?

A) Check if a DFA recognizes the empty set

B) Design a TM that can check if a DFA recognizes the empty set

C) Design a DFA that recognizes the empty set

D) Design a TM that can design a DFA to recognize the empty set

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Will this work?

No! This will run forever if $\langle D \rangle \in \mathrm{E}_{\mathrm{DFA}}$!

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  2. Construct the state graph for D

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B) Design a DFA that accepts at least one string

C) Design a TM that can design a DFA to accept at least one string

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- Note that $L = (\mathrm{E}_{\mathrm{DFA}})^c$
- Decidable languages are closed under complement
- So L is decidable

Decidable DFA Properties

Recap

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- We can design a TM to test if an arbitrary DFA accepts anything at all

Decidable DFA Properties

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- We can design a TM to test if an arbitrary DFA accepts anything at all
- We can design a TM to test if an arbitrary DFA rejects everything

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What should we do to show that L is decidable?

A) Design a TM to check if a given DFA accepts the string 111

B) Design a TM to check if a given DFA accepts the string $x111y$

C) Design a TM to check if a given DFA accepts any string containing 111

D) Design a TM to check if a given DFA only accepts strings containing 111

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Consider the following language:

$$L = \{\langle D \rangle | x111y \in L(D) \text{ for some } x, y \in \Sigma^*\}$$

What should we do to show that L is decidable?

A) Design a TM to check if a given DFA accepts the string 111

B) Design a TM to check if a given DFA accepts the string $x111y$

C) Design a TM to check if a given DFA accepts any string containing 111 ✓

D) Design a TM to check if a given DFA only accepts strings containing 111

Decidable DFA Properties

Let's prove that the following language is decidable

$$L = \{\langle D \rangle | x111y \in L(D) \text{ for some } x, y \in \Sigma^*\}$$

- Hint 1: $\Sigma^*111\Sigma^*$ can be described by a DFA
- Hint 2: Regular languages are closed under intersection
- Hint 3: We can check if a DFA is non-empty

Decidable DFA Properties

Let's prove that the following language is decidable

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Proof Idea: Check that the set of $x111y$ strings that are also accepted by D is not empty

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- Suppose D accepts $x111y$ for some $x, y \in \Sigma^*$

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- Suppose D accepts $x111y$ for some $x, y \in \Sigma^*$
- Then $L(D) \cap (\Sigma^*111\Sigma^*) \neq \emptyset$

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Let's prove that the following language is decidable

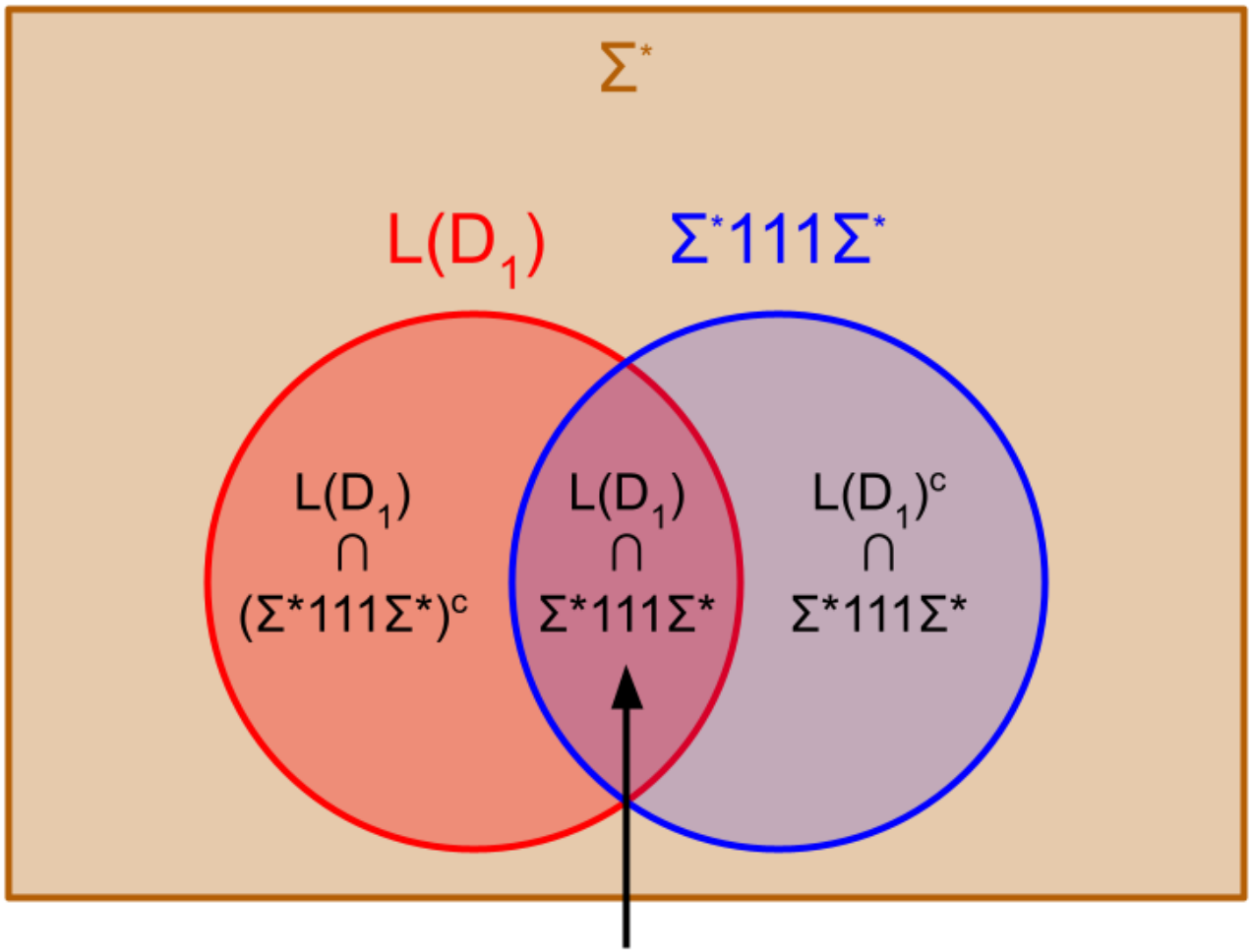
$$L = \{\langle D \rangle | x111y \in L(D) \text{ for some } x, y \in \Sigma^*\}$$

Proof Idea: Check that the set of $x111y$ strings that are also accepted by D is not empty

- Suppose D accepts $x111y$ for some $x, y \in \Sigma^*$
- Then $L(D) \cap (\Sigma^*111\Sigma^*) \neq \emptyset$
  - $x111y$ is accepted by both D and by the regex $\Sigma^*111\Sigma^*$

Decidable DFA Properties

$L = \{\langle D_1 \rangle \mid D_1 \text{ is a DFA, } x111y \in L(D_1) \text{ for some } x, y\}$



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- Let M_E decide $\mathrm{E}_{\mathrm{DFA}}$

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- Design a machine M that decides L

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  2. Create a DFA D_2 that recognizes $\Sigma^*111\Sigma^*$

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  4. Check if $\langle D_3 \rangle \in \mathrm{E}_{\mathrm{DFA}}$

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     - 4.2 Otherwise, M accepts $\langle D \rangle$

This is an example of a machine/program that creates *another* machine/program *at runtime*

Decidable DFA Properties

Prove that the following language is decidable

$$L = \{\langle D \rangle | x111y \in L(D) \text{ for } \underline{\text{all}}\ x, y \in \Sigma^*\}$$

Decidable DFA Properties

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$$L = \{\langle D \rangle | x111y \in L(D) \text{ for } \underline{\text{all}}\ x, y \in \Sigma^*\}$$

- Hint 1: $\Sigma^*111\Sigma^*$ can be recognized by a DFA
- Hint 2: Regular languages are closed under intersection *and complement*
- Hint 3: we can check if a DFA is empty . . .

Decidable DFA Properties

Prove that the following language is decidable

$$L = \{\langle D \rangle | x111y \in L(D) \text{ for } \underline{\text{all}}\ x, y \in \Sigma^*\}$$

Proof Idea: We check that the set of $x111y$ string that are rejected by D is empty

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- We check that $\Sigma^*111\Sigma^*$ has nothing in common with the complement of $L(D)$

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- Suppose $\Sigma^*111\Sigma^* \subseteq L(D)$

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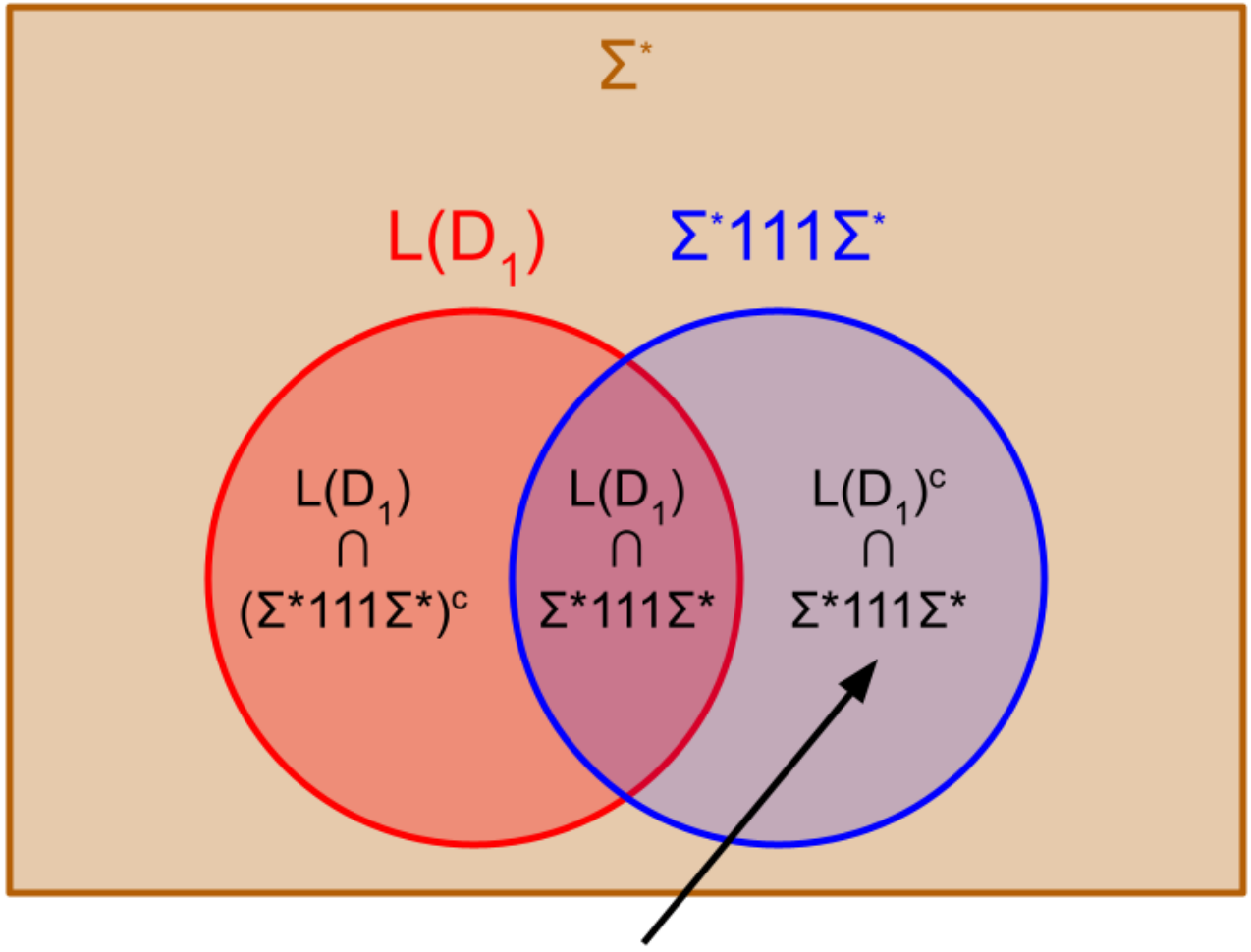
Proof Idea: We check that the set of $x111y$ string that are rejected by D is empty

- We check that $\Sigma^*111\Sigma^*$ has nothing in common with the complement of $L(D)$
- Suppose $\Sigma^*111\Sigma^* \subseteq L(D)$
- Then $L(D)^c \cap (\Sigma^*111\Sigma^*) = \emptyset$

Decidable DFA Properties

$L = \{\langle D_1 \rangle \mid D_1 \text{ is a DFA, } x111y \in L(D_1) \text{ for all } x, y\}$



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Decidability of $\mathrm{EQ}_{\mathrm{DFA}}$

Theorem: The following language is decidable

$$\mathrm{EQ}_{\mathrm{DFA}} = \{\langle D_1, D_2\rangle | L(D_1) = L(D_2)\}$$

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Theorem: The following language is decidable

$$\mathrm{EQ}_{\mathrm{DFA}} = \{\langle D_1, D_2\rangle | L(D_1) = L(D_2)\}$$

- Hint: regular languages are closed under complement, and union
- Hint: try to figure out if there is a string that is accepted by one machine but not the other

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Proof Idea: check that the set of strings that are accepted by one DFA and rejected by the other is empty

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  - No string is in L_1 but not L_2

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- $(L_1 \cap L_2^c) \cup (L_1^c \cap L_2) = \emptyset$

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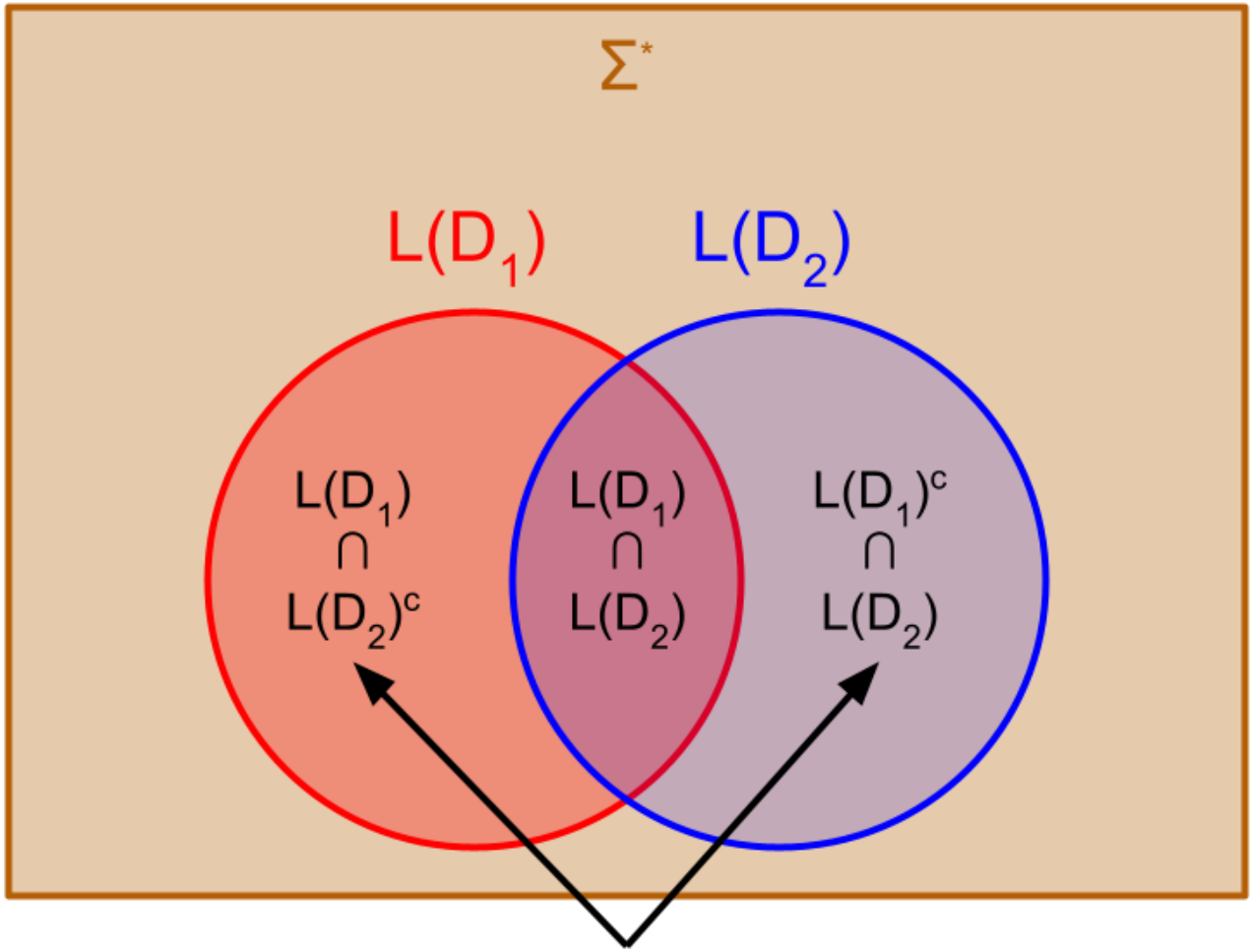
- Suppose $L_1 = L_2$
- Then $L_1 \cap L_2^c = \emptyset$
  - No string is in L_1 but not L_2
- $L_1^c \cap L_2 = \emptyset$
  - No string is in L_2 but not L_1
- $(L_1 \cap L_2^c) \cup (L_1^c \cap L_2) = \emptyset$
  - No string is in one language but not the other

Decidability of $\mathrm{EQ}_{\mathrm{DFA}}$

L = {<D_1, D_2> | D_1, D_2 are DFAs, $L(D_1) = L(D_2)$}



Check that these areas are empty

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  2. Create a DFA D_3 that recognizes $L(D_1) \cap L(D_2)^c$

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  1. M takes $\langle D_1, D_2\rangle$ as input
  2. Create a DFA D_3 that recognizes $L(D_1) \cap L(D_2)^c$
  3. Create a DFA D_4 that recognizes $L(D_1)^c \cap L(D_2)$

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  1. M takes $\langle D_1, D_2\rangle$ as input
  2. Create a DFA D_3 that recognizes $L(D_1) \cap L(D_2)^c$
  3. Create a DFA D_4 that recognizes $L(D_1)^c \cap L(D_2)$
  4. Create a DFA D_5 that recognizes $L(D_3) \cup L(D_4)$

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Decidable DFA Properties

Recap

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- We can test if an arbitrary DFA intersects with or contains a certain collections of strings

Decidable DFA Properties

Recap

- We can test if an arbitrary DFA intersects with or contains a certain collections of strings
- We can test if two DFAs are equivalent

Decidable Turing Machine Properties

Consider the following language

$L = \{\langle M, w\rangle | M \text{ is a TM that halts on w in 100 steps}\}$

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What is input to the decision problem associated with this language?

A) The description of a TM

B) The description of a TM, as well as an input string

C) The description of a TM that runs for 100 steps

D) The description of a TM, as well as a string that it finishes processing in 100 steps

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Decidable Turing Machine Properties

Consider the following language

$L = \{\langle M, w\rangle | M \text{ is a TM that halts on w in 100 steps}\}$

What should we do to show that L is decidable?

A) Design a machine that can check if a given TM halts on a given string in 100 steps

B) Design a machine that can create a machine that runs for 100 steps

C) Design a machine that can accept w within 100 steps

D) Design a machine that can check if a given TM always runs for 100 steps

Decidable Turing Machine Properties

Consider the following language

$L = \{\langle M, w\rangle | M \text{ is a TM that halts on w in 100 steps}\}$

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  4. If M runs for 100 steps without accepting or rejecting, M_L rejects $\langle M, w\rangle$

Decidability of HALT

Prove that the following language is decidable

$$\mathrm{HALT} = \{\langle M, w\rangle | M \text{ halts on } w\}$$

Check if an arbitrary program M finishes processing an arbitrary input w in *any* number of steps

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  4. If M loops on w, M_H...loops on $\langle M, w\rangle$

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- How do we prove such a statement?