Turing Machines

Arjun Chandrasekhar

History of Computer Science

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- Church-Turing Thesis

Turing Machine

Turing Machine

A new model of computation

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- Tape consisting of infinitely many squares

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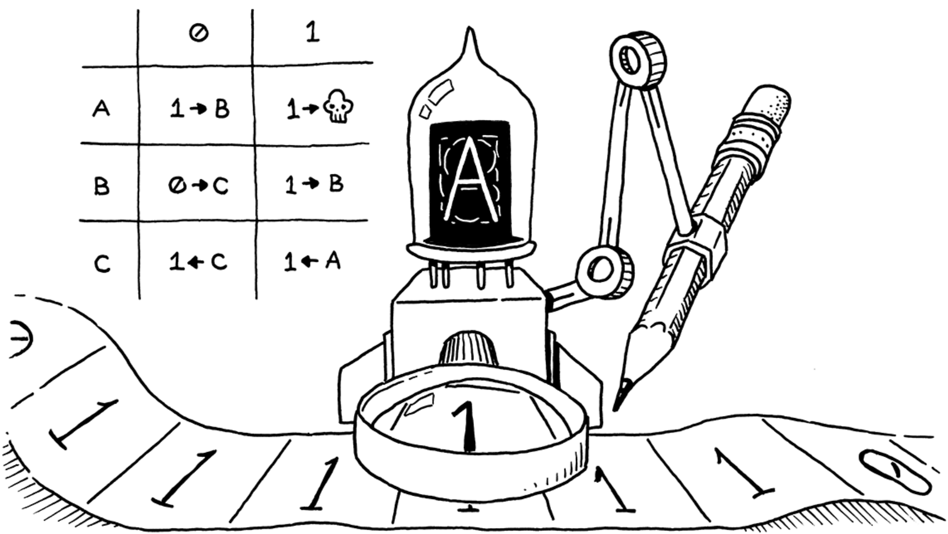
Turing Machine



Turing Machine Formal Description

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Before you get stressed out by the next slide...

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- You do NOT need to memorize the formal definition of a TM

Turing Machine Formal Description

Before you get stressed out by the next slide...

- You do NOT need to memorize the formal definition of a TM
- I will NEVER ask you to give a formal description - informal descriptions will suffice on all assignments and exams

Turing Machine Formal Description

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$(Q, \Sigma, \Gamma, \delta, q_s, q_A, q_R)$

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4. This continues until the machine enters the accept or reject state

Turing Machine Example

A machine to recognize $\Sigma^* 1 \Sigma^*$

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1. Scan left to right

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Turing Machine Example

A machine to recognize $\Sigma^*1\Sigma^*$

1. Scan left to right
2. If we encounter a 1, immediately accept
3. If we encounter a blank space (i.e. end of input), reject

Turing Machine Example

A machine to recognize $\Sigma^*1\Sigma^*$ (formal definition)

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1. $Q = \{q_0, q_A, q_R\}$

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2. $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, \sqcup\}$

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1. $Q = \{q_0, q_A, q_R\}$
2. $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, \sqcup\}$
3. $\delta(q_0, 0) = (q_0, 0, R)$
   $\delta(q_0, 1) = (q_A, 1, R)$
   $\delta(q_0, \sqcup) = (q_R, \sqcup, L)$

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What does this machine do on 010? 00?

Turing Machine Configuration

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- Over time, the TM changes three things: the tape head state, the tape contents, and the tape head location

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- A TM **configuration** is a formal way of describing the overall state of the machine

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  3. The tape head is on the first symbol of v

Turing Machine Configuration

What is the TM state in this configuration?

$$1011q_7 01111$$

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The TM is in state q_7

Turing Machine Configuration

What are the tape contents in this configuration?

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The tape contents are 101101111

Turing Machine Configuration

Where is the TM head in this configuration?

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Turing Machine Configuration

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The TM is on top of the second 0

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Turing Machine Rejection

- The **start configuration** of M on input w is the configuration $q_0 w$
- Any configuration that includes q_R is a **rejecting configuration**
- A Turing Machine M rejects input w if there are a sequence of configurations $C_1, C_2, \ldots, C_n$ where
  1. C_i is the start configuration
  2. Each C_i yields C_{i+1}
  3. C_n is a rejecting configuration

Turing Machine Computation

Some notes

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- Unlike automata, a TM does *not* have to read characters one by one; it starts with the entire input on the tape, and it may move around freely

Turing Machine Computation

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- Unlike automata, a TM does *not* have to read characters one by one; it starts with the entire input on the tape, and it may move around freely
- If the machine is at the left end of the tape and it tries to move left, it stays in place

Turing Machine Example

What does the following TM do on the input ϵ?

State	0	1	$\sqcup$
q_0 (start)	$0 \rightarrow q_1$	$1 \rightarrow q_2$	$\sqcup \rightarrow \mathrm{q}_{\mathrm{reject}}$
q_1	$0 \rightarrow q_1$	$1 \rightarrow q_1$	$\sqcup \rightarrow q_1$
q_2	$0 \rightarrow q_2$	$1 \rightarrow q_2$	$\sqcup \rightarrow \mathrm{q}_{\mathrm{accept}}$

A. Accept

B. Reject

C. Loop

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A. Accept

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Turing Machine Example

What does the following TM do on the input 000?

State	0	1	$\sqcup$
q_0 (start)	$0 \rightarrow q_1$	$1 \rightarrow q_2$	$\sqcup \rightarrow \mathrm{q}_{\mathrm{reject}}$
q_1	$0 \rightarrow q_1$	$1 \rightarrow q_1$	$\sqcup \rightarrow q_1$
q_2	$0 \rightarrow q_2$	$1 \rightarrow q_2$	$\sqcup \rightarrow \mathrm{q}_{\mathrm{accept}}$

A. Accept

B. Reject

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A. Accept

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Turing Machine Example

What does the following TM do on the input 111?

State	0	1	$\sqcup$
q_0 (start)	$0 \to q_1$	$1 \to q_2$	$\sqcup \to \text{q}_{\text{reject}}$
q_1	$0 \to q_1$	$1 \to q_1$	$\sqcup \to q_1$
q_2	$0 \to q_2$	$1 \to q_2$	$\sqcup \to \text{q}_{\text{accept}}$

A. Accept

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What does the following TM do on the input 111?

State	0	1	$\sqcup$
q_0 (start)	$0 \rightarrow q_1$	$1 \rightarrow q_2$	$\sqcup \rightarrow \mathrm{q_{reject}}$
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A. Accept ✓

B. Reject

C. Loop

Turing Machine Example

What does the following TM do on the input 101?

State	0	1	$\sqcup$
q_0 (start)	$0 \to q_1$	$1 \to q_2$	$\sqcup \to \mathrm{q_{reject}}$
q_1	$0 \to q_1$	$1 \to q_1$	$\sqcup \to q_1$
q_2	$0 \to q_2$	$1 \to q_2$	$\sqcup \to \mathrm{q_{accept}}$

A. Accept

B. Reject

C. Loop

Turing Machine Example

What does the following TM do on the input 101?

State	0	1	$\sqcup$
q_0 (start)	$0 \rightarrow q_1$	$1 \rightarrow q_2$	$\sqcup \rightarrow \mathrm{q}_{\mathrm{reject}}$
q_1	$0 \rightarrow q_1$	$1 \rightarrow q_1$	$\sqcup \rightarrow q_1$
q_2	$0 \rightarrow q_2$	$1 \rightarrow q_2$	$\sqcup \rightarrow \mathrm{q}_{\mathrm{accept}}$

A. Accept ✓

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Turing Machine Halting

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A Turing machine **halts** on input w if it accepts or rejects w

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Turing Machine Halting

A Turing machine **halts** on input *w* if it accepts or rejects *w*

- The machine doesn't simply read each character once - it can go back and forth
- THERE IS NO INHERENT GUARANTEE THAT A TURING MACHINE WILL HALT

Turing Machine Descriptions

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There are three different ways we can describe a Turing machine

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1. **High level description**

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1. **High level description**
2. **Tape/implementation level description**
3. **Formal description**

High Level Description

Clear, English description of the algorithm to solve the problem, but does not describe how to specifically implement it on a Turing machine

Tape Level Description

Clear description of how the Turing machine moves around and manipulates the tape, and when it accepts/rejects, but doesn't describe *every* individual state and transition

Formal Description

Describes every single state and every single transition

Turing Machine Example

Let's design a Turing machine to recognize the language of palindromes

$$L = \{w \in \{a, b\}^* | w = w^R\}$$

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1. **Base Case:** if $w = \epsilon$ then accept

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High level description:

1. **Base Case:** if $w = \epsilon$ then accept
2. **Recursive Case:** if $|w| = n \geq 1$, check if the first and last character match. If they do, erase them, and recurse on all the middle characters characters.

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   2.1 If so, erase it, go back to the start and repeat
   2.2 If not, reject immediately
3. Once the tape is blank, accept

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$$L = \{w \in \{a, b\}^* | w = w^R\}$$

Formal description:

	a	b	$\sqcup$
q_s (start)	$\sqcup \rightarrow q_a$	$\sqcup \rightarrow q_b$	$\sqcup \rightarrow \mathrm{q}_{\mathrm{accept}}$
q_a	$a \rightarrow q_a$	$b \rightarrow q_a$	$\sqcup \leftarrow q_a*$
q_b	$a \rightarrow q_b$	$b \rightarrow q_b$	$\sqcup \leftarrow q_b*$
q_a*	$\sqcup \leftarrow q_s*$	$\sqcup \leftarrow \mathrm{q}_{\mathrm{reject}}$	$\sqcup \leftarrow q_s*$
q_b*	$\sqcup \leftarrow \mathrm{q}_{\mathrm{reject}}$	$\sqcup \leftarrow q_s*$	$\sqcup \leftarrow q_s*$
q^s*	$a \leftarrow q_s*$	$b \leftarrow q_s*$	$\sqcup \rightarrow q_s$

The language of a TM

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Let M be a TM. We say the **language of** M, denoted $L(M)$, is the set of strings that are accepted by M

Turing-Decidable Languages

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- With Turing-machines, we have to explicitly distinguish between deciding and recognizing a language.

Turing Machines

Let's show that the following language is Turing-decidable

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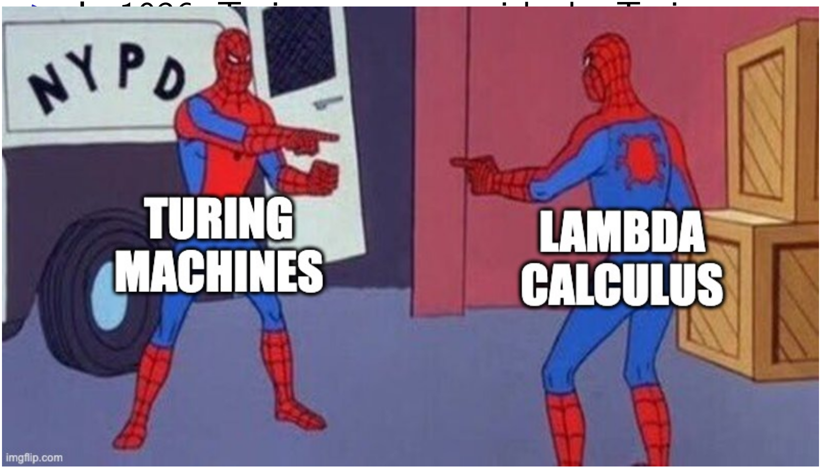

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- Since then, several other models of computation have been proposed, and they all turned out to be equivalent to Turing machines.
    - We have yet to define a machine or programming language that is *more* powerful than a Turing machine

Church-Turing Thesis

The **Church-Turing Thesis** states that Turing machines (and all equivalent models) correspond our intuitive notion of what an "algorithm" is

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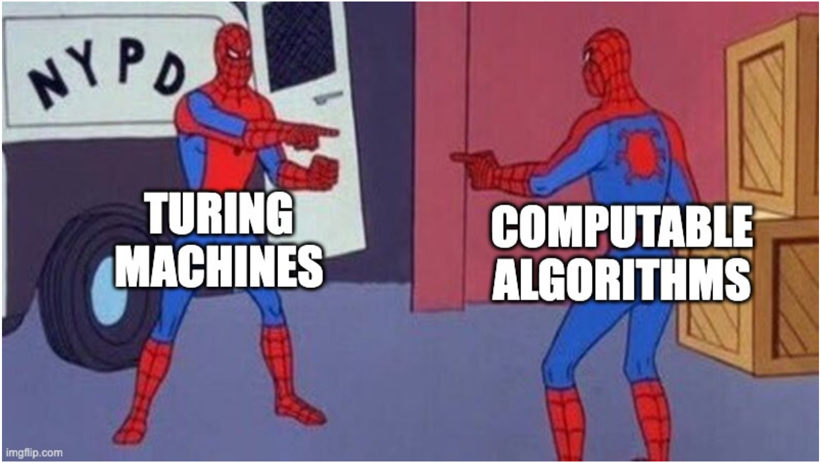

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- Any task that can be solved using a mechanical procedure can be solved using a Turing machine
- This is not a theorem or even a mathematically precise statement - but we accept it as true because we have yet to find any satisfying counterexamples

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- To show that a language can be decided or recognized by a Turing machine, a high-level algorithmic description will suffice
- We can appeal to the Church-Turing thesis so say that whatever algorithm we describe can be implemented on a TM
- No more tedious formal descriptions
- We still use tape-level descriptions to show that a new machine is equivalent to a TM

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By the Church-Turing thesis we can implement this algorithm on a TM. This algorithm will always halt. Thus L is Turing-decidable.

Turing Machine Example

Let's show that $\{a^n b^n c^n | n \geq 0\}$ is Turing-decidable

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- For practice, let's give both a high-level description and a tape-level description

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High level description:

1. Make sure the a's, b's, and c's are in the right order
2. Count the a's, b's, and c's. If they are equal, accept. Otherwise, reject.

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3. If the tape becomes empty, accept.